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ON THE THERMODYNAMIC INTERPRETATION OF ISENTROPIC CHARTS

By HORACE R. BYERS

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The isentropic chart serves a twofold purpose as (1) a hydrodynamic, and (2) a thermodynamic chart—hydrodynamic in the sense that the paths of “tongues” of maximum water-vapor content serve as identifying indicators of the flow pattern and of lateral mixing; and thermodynamic with respect to indications of adiabatic changes in the air, including water vapor, as it flows along the sloping or broadly undulating isentropic surface. The thermodynamic interpretations will be emphasized in this paper.

As introduced into daily synoptic practice by Rossby and collaborators,¹ the chart usually is drawn to represent a contour map of an isentropic surface, with isograms of specific humidity along the same surface superimposed. Other data, such as winds, relative humidities, clouds, etc., are entered at points of observation with no special representation as to areal distribution. For thermodynamic interpretations it is preferable to use *isobars* in place of elevation contours on the charts, and *isentropic-condensation pressures* instead of specific humidities. In this form, which has been developed and applied in daily practice by the writer and his colleagues, the charts have the same general appearance as the specific humidity-altitude charts, because pressure and altitude are closely related and the condensation pressure along an isentropic surface has only one value for each specific humidity; but the use of condensation pressure and actual pressure has important advantages over the use of specific humidity and altitude, which will be outlined in this article.

Definitions and derivations.—The isentropic-condensation pressure is defined as the total pressure to which air containing water vapor in the unsaturated state must be expanded adiabatically or isentropically without gain or loss of moisture in order to reach saturation. In other words, it is the pressure of the condensation level in an ordinary adiabatic process in the atmosphere. Designating it as p_0 , one finds from Poisson's equation that

$$p_0 = p \left(\frac{T_0}{T} \right)^{\frac{mC_p}{R}} \quad (1)$$

where T is the temperature at a point with pressure p ($> p_0$) on the isentropic surface; T_0 is the temperature at p_0 (isentropic-condensation temperature); m , the molecular weight of dry air; C_p , the specific heat at constant pressure (work units), and R , the universal gas constant. The definition of the isentropic-condensation pressure re-

quires that the specific humidity (q) shall remain unchanged during the isentropic process, and, making use of this, one obtains the following relations:

$$q = \beta \frac{e}{p} = \beta \frac{e_0}{p_0} \quad (2)$$

$$\frac{p_0}{p} = \frac{e_0}{e} = \left(\frac{T_0}{T} \right)^{\frac{mC_p}{R}} \quad (3)$$

$$p_0 = p \frac{e_0}{e} = p \left(\frac{T_0}{T} \right)^{\frac{mC_p}{R}} \quad (4)$$

Here e is the partial pressure of the water vapor and β is the ratio of the molecular weight of water vapor to that of dry air, or 0.622, (q in grams per gram).² The values of p_0 , T_0 and q as well as the potential temperature can be obtained from any of the thermodynamic charts commonly used in meteorology, such as the Hertz-Neuhoff (pseudo-adiabatic) diagram, Rossby diagram or the tephigram.³

To demonstrate that p_0 for any given isentropic surface is a function of q alone, it is necessary to introduce two more equations in addition to (2), since the latter contains three variables. Magnus' formula,

$$e_0 = k \epsilon^{\frac{\alpha T_0}{\gamma + T_0}} \quad (5)$$

where ϵ is the base of the natural logarithms and k , α and γ are constants, can be used with the aid of another relation which can be derived from Poisson's equation as follows:

If in (1) $p = 1000$ mb, then $T = \theta$, the potential temperature. Along an isentropic surface, then,

$$\theta = T_0 \left(\frac{1000}{p_0} \right)^{\frac{R}{mC_p}} \quad (6)$$

and

$$T_0 = \frac{\theta}{c} p_0^\kappa \quad (7)$$

where $\kappa = \frac{R}{mC_p}$ and $c = 1000^\kappa$.

¹ Rossby, C. G., and collaborators, Isentropic analysis, *Bul. Am. Meteorol. Soc.*, vol. 18, Nos. 6-7, 1937.

² In (2) the denominator should be $p - 0.375e$, but e is so small in comparison with p that the above simplification is justified.

³ Most of these charts give the mixing ratio rather than the specific humidity, but the difference between these two quantities is insignificant in meteorology.

Substituting this value for T_0 in Magnus' formula (5), one obtains

$$e_0 = k e^{\frac{\alpha \theta p_0^*}{b + \theta p_0^*}} \quad (8)$$

where $b = c \cdot \gamma$, and (2) becomes

$$q = \frac{\beta k e^{\frac{\alpha \theta p_0^*}{b + \theta p_0^*}}}{p_0} \quad (9)$$

which shows that on a given isentropic surface p_0 and q are direct functions of each other.

Some advantages.—An inspection of the Hertz-Neuhoff or similar diagram will show that p_0 -isobars on the isentropic chart are better than isograms of specific humidity for representing significant moisture differences at low values of the specific humidity. Figure 1 demonstrates this. The curves are drawn for several isentropic surfaces defined by the potential temperatures 283, 293, 303 and 313, and constructed in accordance with (9). It will be noted that along each isentropic surface $\left(\frac{\partial p_0}{\partial q}\right)_{\theta \text{ const.}}$

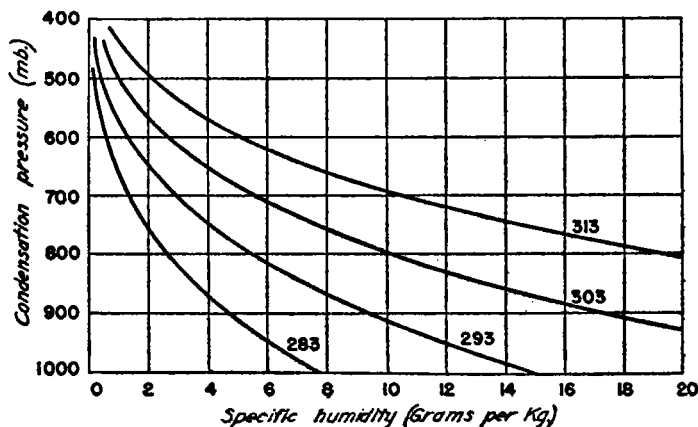


FIGURE 1.

is greatest at low values of q ,⁴ assuring good representation of moisture differences at low values. While on the isentropic chart a difference in specific humidity between 1 and 0.5 g per kg cannot be represented if q -lines are used, the use of p_0 -isobars makes it stand out about as well as the difference between 10 and 5 g/kg. In winter, substantial amounts of precipitation fall out of clouds, in which the specific humidity often is less than 1 g/kg.⁵

The ratio of the partial pressures of the water vapor $\frac{e_0}{e}$, as well as the ratio of the total pressures, $\frac{p_0}{p}$, which in (3) is shown to be an exponential function of the temperature ratio, is a measure of the nearness to condensation. In this sense it is similar to the relative humidity, the latter representing the percentage of saturation with respect to a constant-pressure process while $\frac{e_0}{e} = \frac{p_0}{p}$ measures the fraction of saturation according to an adiabatic or isentropic process. In the analysis of isentropic flow, the ordinary relative humidity has no direct thermodynamic significance. The quantity $\frac{p_0}{p}$ which may be called the "condensation ratio" takes its place.

On the isentropic chart, drawn according to the plan described here, the nearness to condensation is shown by the distance between an isobar and the p_0 -isobar which has the same value. Where these two meet, saturation would be indicated; and unless supersaturation is admitted, they cannot cross. In this way, with a reasonably dense network of aerological stations, such as in the United States, the regions of saturation for any one or several isentropic surfaces can be delineated with accuracy. With specific humidity and elevation lines it is not possible to do this without additional labor, and even then the method would not be accurate because, owing to horizontal pressure gradients, the height of the condensation level corresponding to a given specific humidity would not be everywhere the same.

The distance on the chart separating equal p_0 - and p -isobars is a good prognostic criterion when considered in relation to the prevailing flow pattern. If in extrapolating the flow into the future it is found that p_0 -isobars are overtaking the corresponding p -isobars, saturation and precipitation may be expected to follow. The likelihood of saturation will be indicated especially if the p -gradient $(-\nabla p)$ is greater than $(-\nabla p_0)$.

On the isentropic chart isobars are also isotherms, since $T = \frac{\theta}{c} p^*$ as in (7). Also, it is evident from (7) that p_0 -isobars are isotherms of T_0 , the condensation temperature. Since the relation is not linear, the pressure gradient cannot be used for obtaining the temperature gradient directly. In airways forecasting and dispatching it is helpful to know the location of the 0° isotherm as an indication of possibilities of ice formation, especially where the p - and p_0 -isobars approach each other. It is convenient for this purpose to draw this isotherm in some distinguishing color across the chart. The pressures at the 0° isotherm for various isentropic surfaces is given in the table below.

θ ($^\circ\text{A}$)	p (mb)	θ ($^\circ\text{A}$)	p (mb)	θ ($^\circ\text{A}$)	p (mb)
273	1,000	291	801	309	650
275	975	293	782	311	636
277	950	295	764	313	622
279	928	297	746	315	608
281	905	299	729	317	595
283	883	301	712	319	582
285	861	303	696	321	570
287	841	305	680	323	558
289	820	307	665		

Isobars are also isosteres and isopycnals (lines of equal specific volume and density) on the isentropic surface. This is evident from the following relationships in which v represents the specific volume:

$$T = \frac{\theta}{c} p^* = \frac{p v m}{R} \quad (10)$$

$$v = \frac{R}{m} \frac{\theta}{c} p^{*-1} \quad (11)$$

The gradient of v is a function of the logarithm of the pressure gradient.

An advantage of isobars over elevation contours is seen in the analysis of the isentropic chart in the region of cold cyclones, which are important features of the circulation in middle to high latitudes. On account of the low temperature, these cyclones are intense at high levels and therefore produce a pronounced pressure minimum

⁴ The equation for the derivative is somewhat complicated and difficult to interpret.

⁵ See Lichtblau, S.: Upper-air cold fronts in North America, MONTHLY WEATHER REVIEW, vol. 64, pp. 414-425, 1937.

on the upper as well as the lower isentropic surfaces. The low temperature further contributes to an elevation of these surfaces toward low pressures. In terms of altitude, there is no significant effect due to the low pressure. The height of a given surface depends on the temperature in the air column below and is very little affected by the actual values of the pressures. Besides temperature, the difference in the logarithms of the pres-

be much greater in terms of isobars than in terms of elevation contours. This suggests a considerable amount of solenoidal circulation, which is more to be expected in these cyclones than the rather weak activity suggested by elevation contours.

Due to horizontal pressure gradients a level surface may not be a surface of constant pressure. Furthermore, the isentropic surface may slope in terms of elevation but

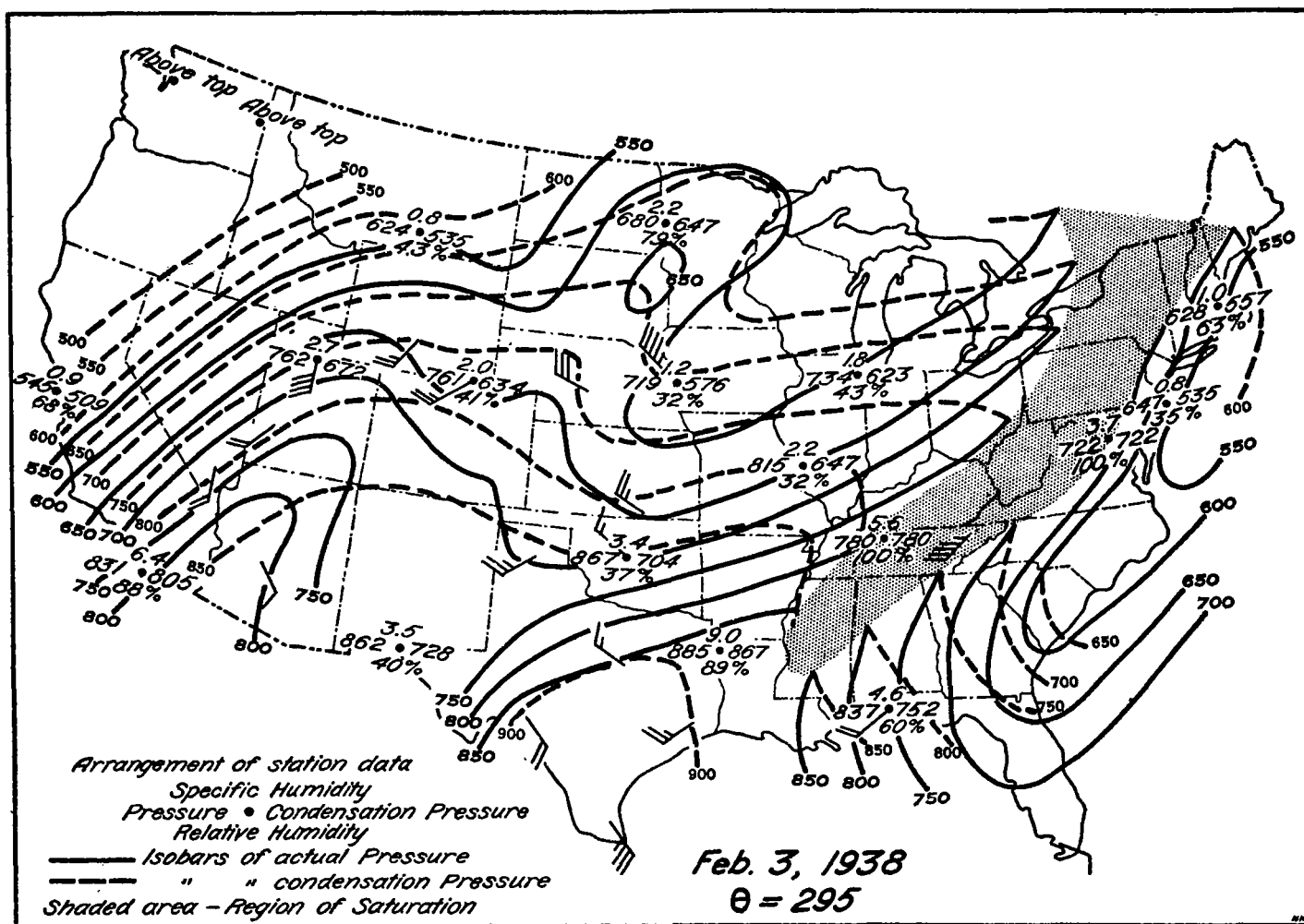


FIGURE 2.

sures at sea level and at the isentropic surface is the only other variable. The height is given by

$$h = \frac{RT_m}{mg} (\log p_s - \log p) \quad (12)$$

where T_m is the integrated-mean temperature of the air column and p_s is the pressure at sea level. As an extreme example of the effect of actual pressures on height at a given level, consider a change in p_s and p of 80 mb each. Then for p_s near 1,000 mb and p near 600 mb the change in altitude at a constant mean temperature of 273° A would be only about 40 meters. Thus, regardless of whether the pressure field shows a HIGH or a LOW, the height of the isentropic surface is hardly affected. In terms of pressure, on the other hand, the transition from a high-pressure area to a low-pressure area on an isentropic surface is marked by a slope even if no temperature difference exists. If there is a temperature difference such that the low-pressure area is colder, the slope would

still be at the same pressure. This and the fact that thermodynamic changes on an isentropic surface really depend only on pressure changes, show that pressure is of more fundamental significance than elevation.

Another great advantage lies in the ease with which the weight of air contained between two isentropic surfaces may be determined. For a vertical cross section of unit area this is given by:

$$\int_{h_1}^{h_2} \rho g dh = - \int_{p_1}^{p_2} dp = p_1 - p_2 \quad (13)$$

where ρ is the density, g is the gravity acceleration and h measures the height. Thus with two isentropic surfaces charted, the mass contained between them is given by a simple subtraction of the pressures at the various points. The change in this difference in a stated period gives the isallobaric contribution of the layers in question.

Examples of charts prepared after the new manner are contained in figures 2 and 3 which represent two surfaces

corresponding to potential temperatures of 295° and 299° for February 3, 1938. The p_0 -isobars are drawn with solid lines and the broken lines are the p -isobars. The stippled area represents the region of saturation, found from the points of intersection of the significant isobars, all of which are clearly shown. This area agrees closely with the area of altostratus clouds and precipitation as shown by the surface map. Even without the help of surface reports it is evident that the chart cannot be drawn in any way save to represent the saturation area practically as shown. Another feature of the charts is the moist tongue extending into Minnesota, apparently with a moisture maximum near Fargo and a dry area near Omaha. This feature would hardly show up on the specific humidity and altitude type of chart.

Construction of cross sections.—A very helpful link for aiding in the interpretation between the surface map and

Lines of equal condensation temperature show the moisture content of the air. These have similar advantages over isograms of specific humidity as do isentropic-condensation pressures. In the first place, the isentropic-condensation temperature along a given isentropic surface is a direct function of q . This is shown by substituting (5) for e_0 in (2), for from (7) we have $p_0 = \left(\frac{c}{\theta} T_0\right)^{1/\kappa}$, which, substituted in (2), gives

$$q = \frac{\beta k \epsilon \gamma + T_0}{\left(\frac{c}{\theta} T_0\right)^{1/\kappa}} \quad (14)$$

However, where, as in a cross section, several different potential temperatures are represented, T_0 =isotherms

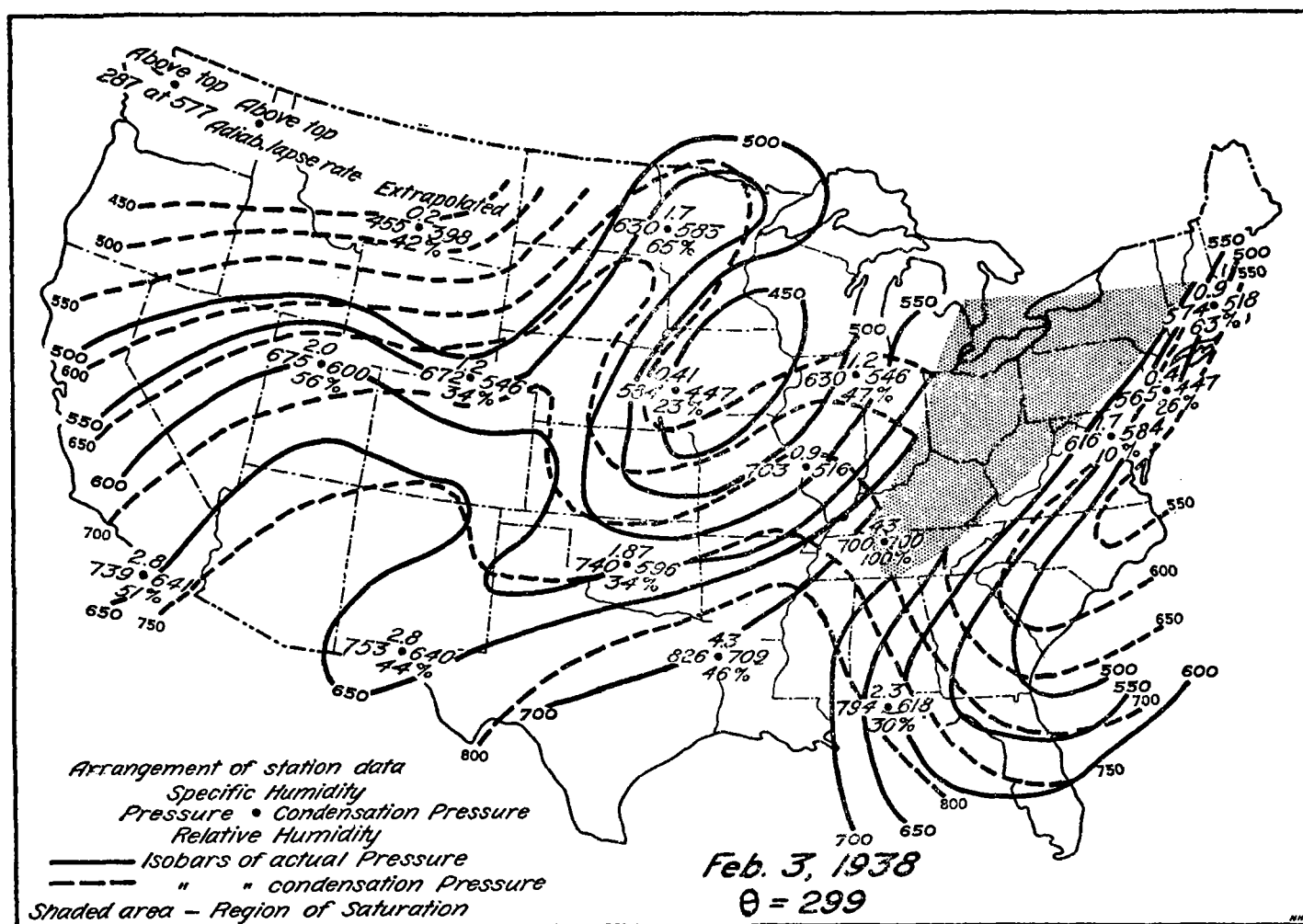


FIGURE 3.

the isentropic chart is the atmospheric cross section. My colleague, C. H. Pierce, has found that for the best thermodynamic interpretation these cross sections should be constructed in terms of altitude and distance, with isotherms of potential temperature and isotherms (in °A) of the isentropic-condensation temperature. Isotherms of potential temperature are, of course, isentropic surfaces. These lines show the slope of the isentropic surfaces in the vertical plane of the cross section constructed, and they also give one some idea of the vertical stability between any two chosen surfaces.

would not correspond to q -isograms because for a given condensation temperature the specific humidity would be different for different pressures. This variation of q with p at constant temperature can be shown by differentiating (2), holding T_0 constant, which, according to (5) means e_0 constant.

$$\left(\frac{\partial q}{\partial p_0}\right)_{T_0 \text{ const.}} = -\beta \frac{e_0}{p_0^2} = -\frac{q}{p_0} \quad (15)$$

Using ordinary values of q and p_0 one finds that this variation in the specific humidity represented by a given T_0

for changing pressure usually is less than 10 percent per 100 mb change. For example, if q is 10 g/kg and p_0 is 1,000 mb, the change would be a 1-gram increase in q per 100 mb ascent in the atmosphere; 0.5 g per 100 mb at a pressure of 500 mb, etc. At 1 g/kg and 1,000 mb the change would be 0.1 g per 100 mb. It is felt that the advantages of using this quantity outweigh this inexactness in the absolute measurement of moisture content. There seems to be no reason why representation in terms of specific humidity should be preferable *per se*, as both are invariant in an adiabatic process.

By comparing the isotherms of potential temperature and the T_0 -isotherms, one can get a fairly good idea of the nearness to condensation. The difference $\theta - T_0$ at a

m to the 2,000 m. Assuming that the airflow is from Omaha to Chicago, one would expect saturation slightly to the east of Chicago at 2,180 m above sea level.

Like the condensation pressure, T_0 gives a more representative difference in moisture content at low values. Furthermore, there would be no crowding of these lines when the moisture content is high, as is the case when isograms of specific humidity are used.

Cross sections of this type, it is to be noted, are constructed entirely of isotherms just as the construction of isentropic charts advocated here consists entirely of isobars.

The moist-adiabatic process.—The ordinary isentropic chart does not give a correct picture of the flow pattern for

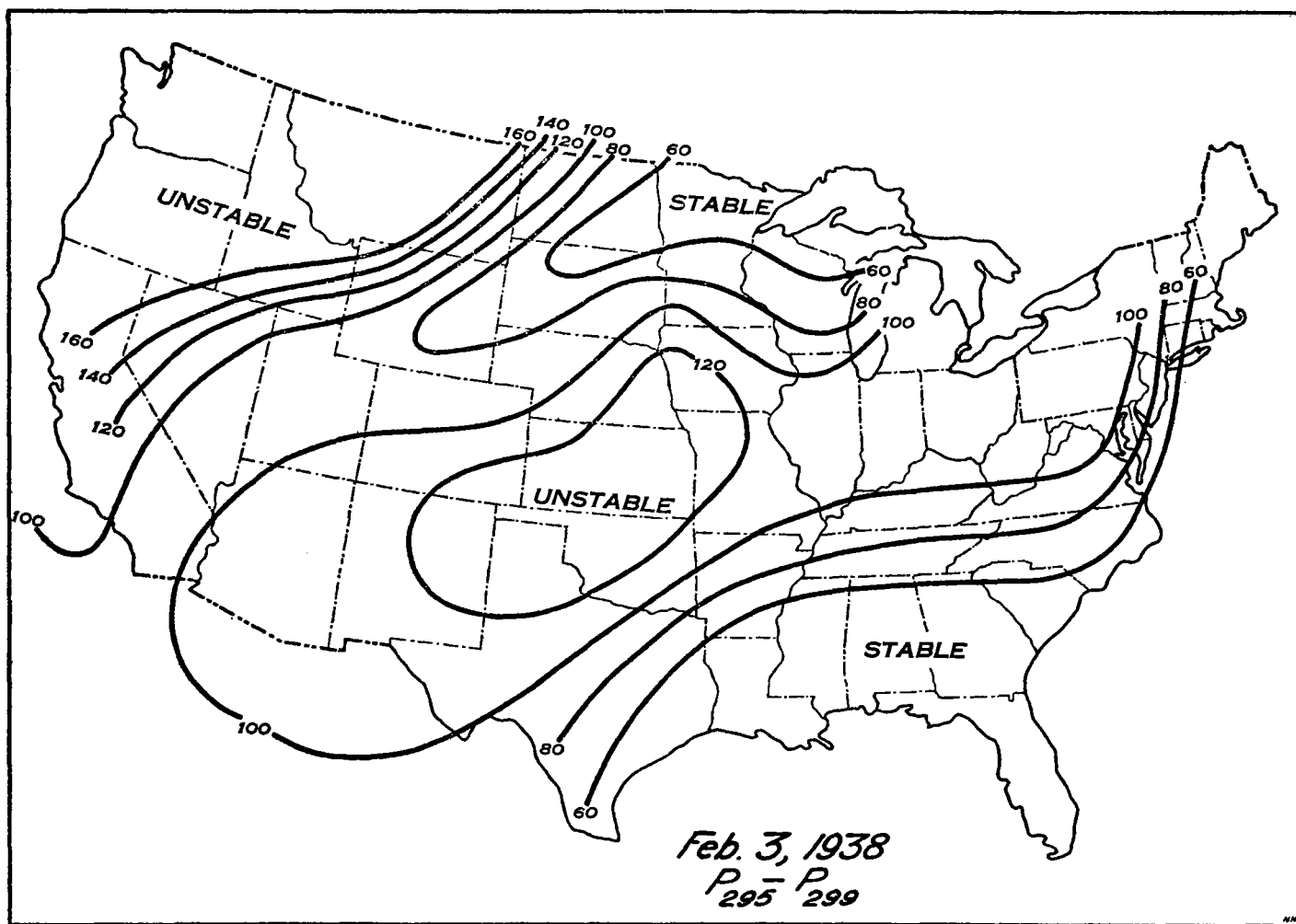


FIGURE 4.

point on the cross section shows the number of degrees the air must be cooled to reach condensation if lifted from the 1,000-mb level. It is known that during adiabatic ascent the temperature falls approximately 10° per km. Therefore, if the difference is 25° , saturation would be expected near 2,500 meters; that is, assuming that the sea-level pressure is 1,000 mb, which is seldom the case. However a rough estimate of the height of the 1,000-mb isobar can be made from the pressures of the surface map.

As an illustration, suppose that at 1,000 m over Omaha we find $\theta = 295$ and $T_0 = 275$, making $\theta - T_0 = 20^\circ$. Accordingly one would expect saturation at 2,000 meters above the 1,000-mb surface. If the 295° isentropic surface slopes upward to 2,000 meters toward Chicago and if the sea-level pressure at Chicago is 1,020 mb we add 180

conditions of saturation. After condensation the air follows a saturation-adiabatic or pseudoadiabatic surface with continuously increasing potential temperature. In this process the equivalent-potential temperature is constant. At the condensation level the pressure, p_0 , and the potential temperature designate the equivalent-potential temperature surface or pseudoadiabatic which subsequently is to be followed. Since after saturation $p_0 = p$, it is easy to find from thermodynamic charts the rate at which this pseudoadiabatic cuts through the successive potential-temperature (isentropic) surfaces. It is not sufficient, however, to calculate in this way the rate of ascent of the air using the vertical and geographical distribution of potential temperature in the surroundings because condensation immediately alters the field of po-

tential temperature. The release of the heat of vaporization from the water vapor upon condensation at any point causes an increase of potential temperature at that point. Thus a lowering of the potential temperature surfaces in the saturated layers occurs.

The transformation from unsaturated to saturated conditions in a region on the isentropic chart is marked by a decrease in the slope of the potential-temperature surfaces. The moist tongues usually flow up a "valley" of the isentropic surface and the effect of saturation in such a "valley" is to change its shape by this decrease in slope.

With a dense network of upper-air soundings in the moist region one would expect to be able to trace the flow in the saturated layers of ascending air by means of the surfaces of equivalent-potential temperature. These would always have a greater slope than the potential-temperature surfaces in the region of saturation. After some experience in constructing isentropic charts where saturation was indicated, it was realized that the lapse rate in the air in the region of saturation should be almost exactly the pseudoadiabatic as a result of stirring within the air layers in question, producing constant equivalent-potential temperature in the vertical. Therefore identification of the surface of constant equivalent-potential temperature is not possible because it is not a surface, but a layer of finite thickness.

The analysis of the two isentropic charts for February 3, 1938 is shown in figures 2 and 3, and the pressure differences between the 295 and 299 surfaces are plotted in figure 4. It will be noted that in the region of saturation the pressure difference is about 100 mb. This is exactly the difference corresponding to a constant equivalent-potential temperature vertically in the layer.

It is possible to carry these ideas farther and state that if the pressure difference between the two surfaces is less than that indicated for constant equivalent-potential temperature (lapse rate less than pseudoadiabatic) it is possible to trace the constant equivalent-potential temperature surface along which the air would move after saturation. However, it is probable that in cases of prolonged cloudiness and precipitation the lapse rate is close to the pseudoadiabatic, a fact which seems to be borne out by observation. Accordingly, no matter what the value of the pressure difference between isentropic surfaces may be in a given region before saturation, after saturation it will soon become equal to that corresponding to a constant equivalent-potential temperature. Further investigations into these various possibilities are contemplated.

Conclusion.—The thermodynamic isentropic chart has the following advantages over the chart based on elevations and specific humidities:

1. Better representation of significant moisture differences at low values. Moist tongues at low temperatures show up as well as those at high temperatures; not so on the other type of chart where with winter temperatures over the continent they may not even appear at all.
 2. Nearness to condensation can be determined by the "condensation ratio" p_0/p directly from the chart and especially by noting on the chart the distance separating a p_0 -isobar and a p -isobar of equal value. This cannot be shown directly with q -isograms. The ratio p_0/p is the only expression of the nearness to saturation that is of direct thermodynamic significance in an isentropic process.
 3. Areas of saturation are indicated directly.
 4. A method of indicating the flow under saturation-adiabatic conditions is provided.
 5. Isotherms can be indicated easily, a given temperature being everywhere at the same pressure. This is useful for aeronautical purposes in indicating the location of the freezing isotherm.
 6. The weight of air in the layers between isentropic surfaces can be read off immediately.
 7. Cold lows appear as regions of greater activity and warm highs as less active than on the other type of chart.
 8. The greater ease of preparation is a distinct advantage. Pressures can be read off directly from the adiabatic chart without the labor of constructing an altitude curve.
- While the thermodynamic uses of the chart have been indicated principally in this paper, it should be noted that advantages listed under 1, 4, 6, and 7 above also accrue in the use of the chart for strictly hydrodynamical purposes. There appears to be no reason why the type of chart advocated here is not preferable for practically all uses, and if only one chart is drawn in daily synoptic practice the one recommended here is distinctly preferable.
- It might be argued that the use of p_0 -isobars does not permit a good measure of the transport by horizontal turbulence, which is proportional to the gradient of specific humidity, $-\nabla q$. At saturation this latter relationship will not hold because there can be no net transport horizontally in this stage; yet $-\nabla q$ will not be zero except along an isobaric surface. In daily synoptic practice an exact computation of the horizontal transport usually is not attempted, although there is no reason why it should not be possible to compute it with a fair degree of reliability. Unless it can be shown that the horizontal transport can be expressed in terms of p_0 in place of q , the method advocated here will have therefore a disadvantage in this one respect.

RELATION OF PRESSURE TENDENCIES TO CYCLONES AND FRONTS

By W. R. STEVENS

[Weather Bureau, New Orleans, La., February 1938]

One of the most important elements for forecasting purposes contained in weather reports is the 3-hour pressure tendency. The method developed by Petterssen¹ for determining displacements of pressure centers, wedges, troughs, and fronts, is one of the most valuable tools at the disposal of the forecaster and is based largely on 3-hour pressure tendencies.

It is generally recognized that tendencies are valuable in analysis as well as in prognosis. To illustrate their value, the writer has prepared a set of model moving cyclones, with and without frontal structures. Attention

is invited to a study in which Bowie² used the same general method in connection with 12-hour pressure changes.

It is emphasized that the 3-hour pressure tendency is only one of many criteria for recognition of a front, and that other data must be given proper consideration.

Each diagram indicates the position of a cyclonic pressure system at the time of observation, as well as 3 hours previously, with attendant pressure changes and characteristics. The changes are larger than ordinarily observed in a 3-hour period, but this is unimportant since they may be divided by any number without affecting the characteristic, which is really the most important factor in analysis.

¹ Petterssen, S.: Kinematical and Dynamical Properties of the Field of Pressure with Application to Weather Forecasting. Geofysiske Publikasjoner vol. X, No. 2.

² Bowie, E. H.: On Pressure-Change Charts, Mo. Wea. Rev., 44: 132-133.